Resilient Monotone Submodular Maximization

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with

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Problems in facility location, machine learning, control

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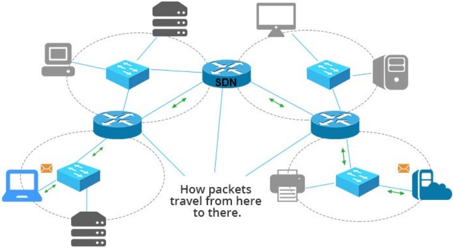
Facility location:
Router placement problem

Goal: Maximize controllable traffic flow in internet service provider networks by replacing legacy routers with SDN routers.



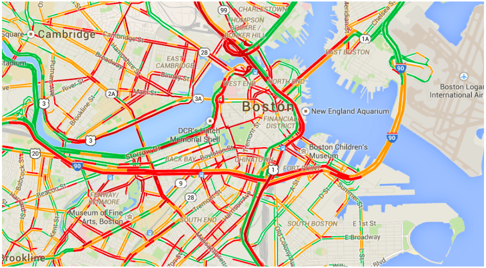
Complication: SDN routers can be expensive.
Problem: Where to place **few** SDNs to achieve goal?[1]

[1] Poularakis et al. '17, IEEE INFOCOM Best paper Award

Problems in facility location, machine learning, control

Machine learning:
Data selection problem

Goal: Maximize prediction accuracy of car traffic by using data collected from cameras and from driver's smart-phones apps.



Complication: Cannot process all data from the data flood.
Problem: What is the **sparsest** data set that achieves goal?[1]

[1] Krause and Guestrin, JMLR '08

Problems in facility location, machine learning, control

Control:
Sensor selection problem

Goal: Maximize quadrotor's localization accuracy by using on-board sensors.

Complication: Quadrotor's battery is limited and sensors need it.
Problem: Which **few** sensors to activate to achieve goal?[1]

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[1] Tzoumas, Carlone, Pappas, Jadbabaie, arXiv: 1709

[2] Additional sensor/actuator selection contributors: Bushnell; Bullo; Clark; Cortes; Jovanovic; Krause; Le Ny; Mo; Olshevsky; Pasqualetti; Pequito; Poovendran; Roy; Sinopoli; Siami; Smith; Summers; Sundaram; Zampieri; Zhang; ...

Problems in facility location, machine learning, control

All previous are monotone submodular maximization problems

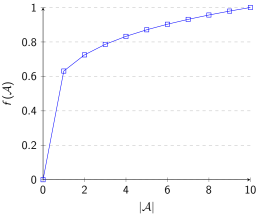
Monotone submodular maximization:

Given:

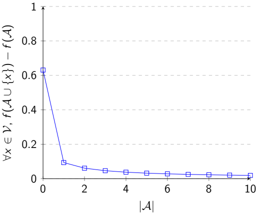
- finite ground set $\mathcal{V}$;
- set function f
- budget α,

solve:

non-decreasing:



submodular:



$$
\begin{aligned}
\max_{\mathcal{A} \subseteq \mathcal{V}} \quad & f(\mathcal{A}) \\
\text{s.t.} \quad & |\mathcal{A}| \leq \alpha.
\end{aligned}
$$

Sensors fail; routers get attacked; data get deleted

if we pick $\mathcal{A}$ to $\max_{\mathcal{A}\subseteq\mathcal{V},|\mathcal{A}|\leq\alpha} f(\mathcal{A})$ → and later a $\mathcal{B} \subseteq \mathcal{A}$ gets deleted → we end up with $f(\mathcal{A}-\mathcal{B})$

Sensors fail; routers get attacked; data get deleted

Denial of Service in Sensor Networks

Unless their de
sensor networ
to denial-of-se

Sensor netwo
tating large-s
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applications
military, env
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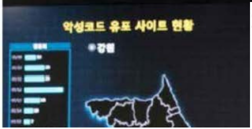
THE WALL STREET JOURNAL.

TECH

New Threats Fuel Fears of Another Global Cyberattack

A new attack hit thousands of computers and a hacking group said it would release more attack softwa



Council of the European Union

Brussels, 11 June 2015 (OR. en)

[...] A natural person should have the right that their personal data are erased and no longer processed [...]

if we pick $\mathcal{A}$ to ~~$\max_{\mathcal{A}\subseteq\mathcal{V},|\mathcal{A}|\leq\alpha} f(\mathcal{A})$~~ → and later a $\mathcal{B} \subseteq \mathcal{A}$ gets deleted → we end up with $f(\mathcal{A} - \mathcal{B})$

Resilient Monotone Submodular Maximization

Problem

Given:

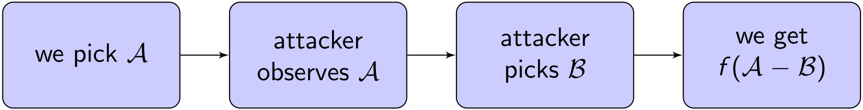
- *finite ground set* $\mathcal{V}$;
- *set function* f *s.t. non-decreasing, submodular,* $f \geq 0$, $f(\emptyset) = 0$;
- *budgets* α, β *s.t.* $0 \leq \beta \leq \alpha \leq |\mathcal{V}|$,

solve:

$$\max_{\mathcal{A} \subseteq \mathcal{V}, |\mathcal{A}| \leq \alpha} \; \min_{\mathcal{B} \subseteq \mathcal{A}, |\mathcal{B}| \leq \beta} \; f(\mathcal{A} - \mathcal{B}).$$

Symbol explanation:

- α: selection budget for resiliency;
- β: maximum number of (future) deletions.



Resilient Monotone Submodular Maximization

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solve:

$$\max_{\mathcal{A} \subseteq \mathcal{V}, |\mathcal{A}| \leq \alpha} \ \min_{\mathcal{B} \subseteq \mathcal{A}, |\mathcal{B}| \leq \beta} f(\mathcal{A} - \mathcal{B}).$$

Difficulties:

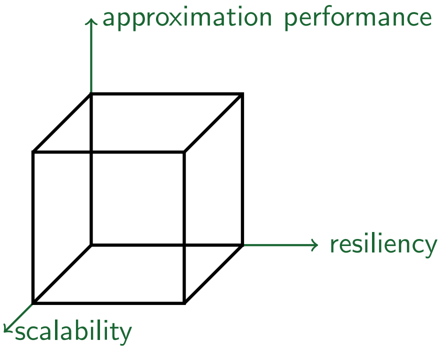
- Problem is NP-hard;[1]
- Function $g(\mathcal{A}) \triangleq \min_{\mathcal{B} \subseteq \mathcal{A}, |\mathcal{B}| \leq \beta} f(\mathcal{A} - \mathcal{B})$ is non-submodular

  $\Rightarrow$ Greedy alg. on $\max_{\mathcal{A} \subseteq \mathcal{V}, |\mathcal{A}| \leq \alpha} g(\mathcal{A})$ can perform arbitrarily bad.[2]

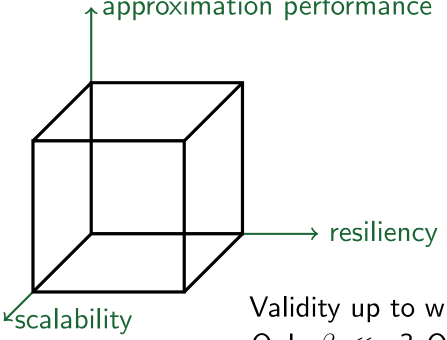
[1] Orlin et al., IPCO '16

[2] Krause et al., JMLR '08

Characteristics of good algorithm



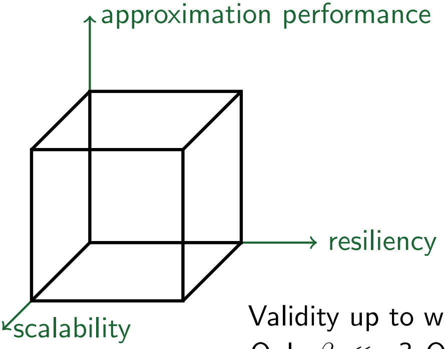
Characteristics of good algorithm



Validity up to what number β of attacks?

Only $\beta \ll \alpha$? Or any $\beta \leq \alpha$?

Characteristics of good algorithm



Validity up to what number β of attacks?

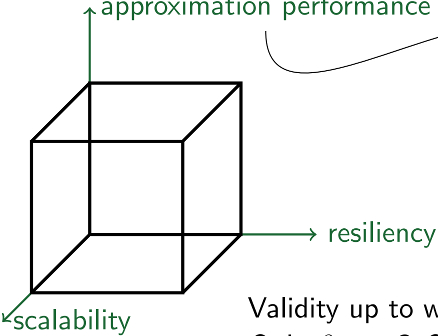
Only $\beta \ll \alpha$? Or any $\beta \leq \alpha$?

low resiliency — high resiliency

Running time w.r.t. α, β, $|\mathcal{V}|$? Exponential? Or linear?

slow — fast

Characteristics of good algorithm



f_{approx} finitely close to f_{opt}?

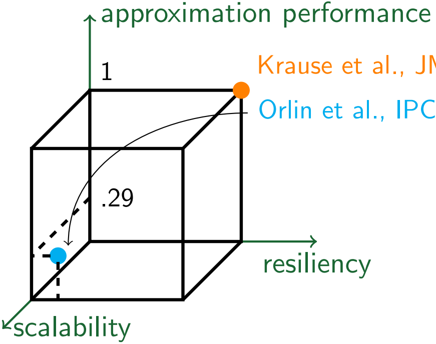
$\underbrace{\frac{f_{\text{approx}}}{f_{\text{opt}}} \geq 0?}_{\text{no suboptimality guarantee}}$ Or $\underbrace{\frac{f_{\text{approx}}}{f_{\text{opt}}} = 1?}_{\text{exact}}$

Validity up to what number β of attacks?

$\underbrace{\text{Only } \beta \ll \alpha}_{\text{low resiliency}}$? Or $\underbrace{\text{any } \beta \leq \alpha}_{\text{high resiliency}}$?

Running time w.r.t. α, β, $|\mathcal{V}|$? $\underbrace{\text{Exponential}}_{\text{slow}}$? Or $\underbrace{\text{linear}}_{\text{fast}}$?

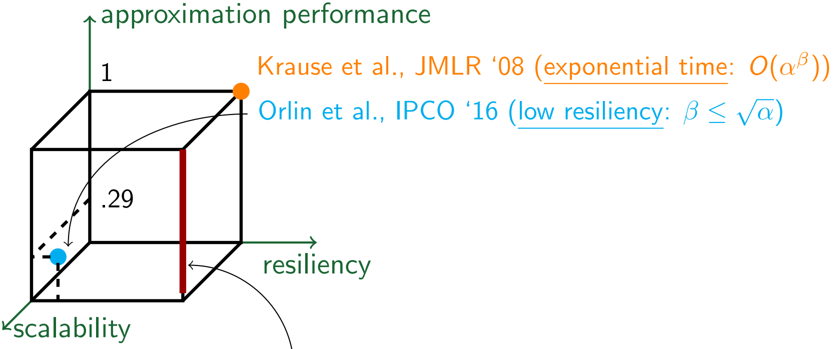
Literature review



Krause et al., JMLR '08 (exponential time: $O(\alpha^{\beta})$)

Orlin et al., IPCO '16 (low resiliency: $\beta \leq \sqrt{\alpha}$)

Literature review

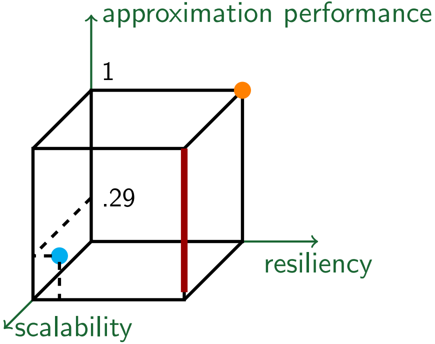


What we ask for: feasibility of red segment

Existence of algorithm that has:

- *High resiliency:* valid for any number β of attacks;
- *High scalability:* running time at most linear in $\alpha, \beta, |\mathcal{V}|$;
- *Provable approximation performance:* non-zero suboptimality guarantee.

Literature review



First claim in the CDC paper

First algorithm that has:

- *High resiliency:* valid for any number β of attacks;
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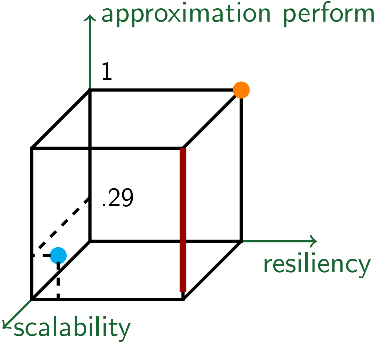
Literature review

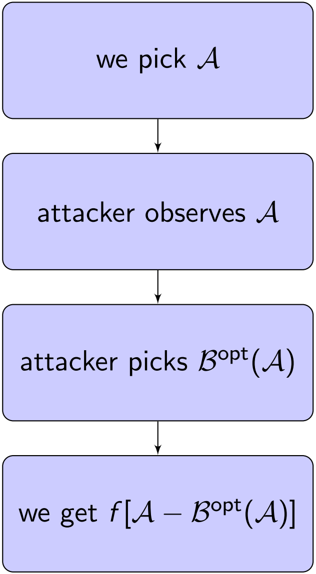


Second claim in the CDC paper

First algorithm that has:

- *High resiliency:* valid for any number β of attacks;
- *High scalability:* running time at most linear in $\alpha, \beta, |\mathcal{V}|$;
- **Superior approximation performance***:* For functions f with low curvature, first algorithm with approximation performance $\geq .29$.

Algorithm


we pick $\mathcal{A}$
attacker observes $\mathcal{A}$
attacker picks $\mathcal{B}^{\text{opt}}(\mathcal{A})$
we get $f[\mathcal{A} - \mathcal{B}^{\text{opt}}(\mathcal{A})]$

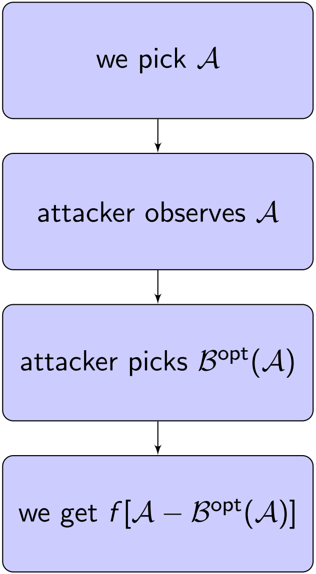
Algorithm

Idea:

Pick $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$

"bait"
$|\mathcal{A}_1| = \beta$.

$\mathcal{A}_2 \subseteq \mathcal{V} - \mathcal{A}_1$;
$|\mathcal{A}_2| = \alpha - \beta$.



Algorithm

Order $\mathcal{V} = \{z_1, \ldots, z_{|\mathcal{V}|}\}$ s.t.: $f(z_1) \geq \ldots \geq f(z_{|\mathcal{V}|})$

↓

Pick bait $\mathcal{A}_1 = \{z_1, \ldots, z_\beta\}$

↓

Pick $\mathcal{A}_2$ greedily from $\mathcal{V} - \mathcal{A}_1$

↓

Return $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$

Idea:

Pick $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$

- $\mathcal{A}_1$: "bait"; $|\mathcal{A}_1| = \beta$.
- $\mathcal{A}_2$: $\mathcal{A}_2 \subseteq \mathcal{V} - \mathcal{A}_1$; $|\mathcal{A}_2| = \alpha - \beta$.

Algorithm's performance

Definition:

f's curvature is defined as:

$$\kappa_f \triangleq 1 - \min_{z\in\mathcal{V}} \frac{f(\mathcal{V})-f(\mathcal{V}\setminus\{z\})}{f(z)}.$$

Properties:

- Computable in $O(|\mathcal{V}|)$ time;
- $0 \leq \kappa_f \leq 1$.

Interpretation:

κ_f measures how $\mathcal{V}$'s elements *substitute* each other:

- $\kappa_f = 0 \Leftrightarrow f(\mathcal{A}) = \sum_{z\in\mathcal{A}} f(z)$;
- $\kappa_f = 1 \Leftrightarrow$ there exist $z \in \mathcal{V}$ s. t. $f(\mathcal{V}) = f(\mathcal{V} \setminus \{z\})$.

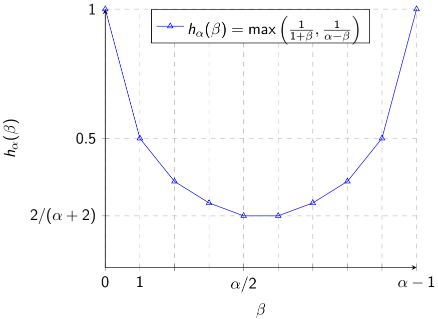
Algorithm's performance

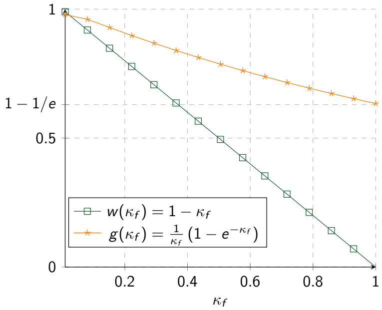
Theorem

Algorithm:

- *(Resiliency) is valid for any* $\beta \leq \alpha \leq |\mathcal{V}|$*;*
- *(Scalability) runs in* $O[(\alpha - \beta)|\mathcal{V}|]$ *time;*
- *(Provable approximation performance) guarantees:*

$$\frac{f_{approx}}{f_{opt}} \geq \max\left[h_\alpha(\beta), w(\kappa_f)\right] g(\kappa_f).$$





Algorithm's performance

Theorem

Algorithm:

greedy algorithm's guarantee for $\max_{\mathcal{A}\subseteq\mathcal{V},|\mathcal{A}|\leq\alpha} f(\mathcal{A})$

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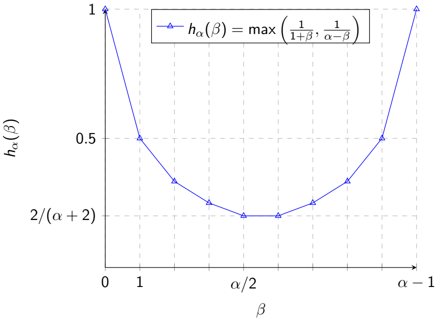
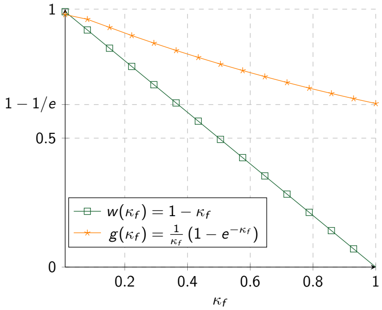




Classes of functions with $\kappa_f < 1$ and applications

- Concave over modular functions;
  - (Machine learning) Image segmentation, speech processing.
- Functions of the form $f(\mathcal{A}) = \log\det(\sum_{i \in \mathcal{A}} D_i + I)$;
  - (Machine learning) Experiment design; feature, data selection.
  - (Control) Sensor and actuator selection.

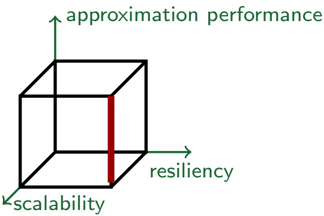
  Example: Gaussian processes with RBF kernels:[3] $\kappa_f \simeq 0$.[4]
- Functions of the form $f(\mathcal{A}) = \text{trace}[(\sum_{i \in \mathcal{A}} D_i + I)^{-1}]$.
  - (Machine learning) Experiment design; feature, data selection.
  - (Control) Sensor and actuator selection.

[3] RBF kernels model physical phenomena such as temperature in buildings.
[4] Sharma et al., ICML '15.

Summary of results

First algorithm that has:

- *High resiliency:* valid for any number β of attacks;
- *High scalability:* running time at most linear in $\alpha, \beta, |\mathcal{V}|$;
- *Provable approximation performance:* suboptimality guarantees.
- *Superior approximation performance:* For curvature values $\kappa_f \leq .71$, first algorithm with approximation performance $\geq .29$.



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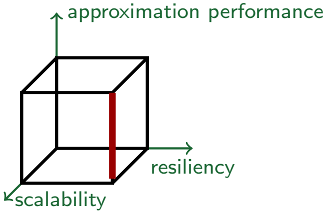
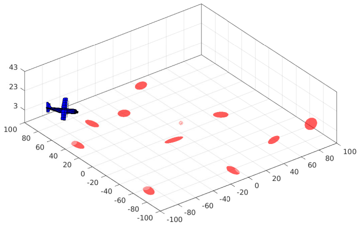
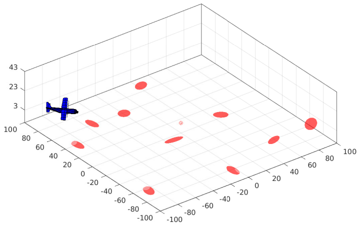


Simulations: robot localization scenario



Scenario: UAV moves in a 3D space.
UAV's model: double-integrator, with state $x_k = [p_k, v_k]^\top$ s.t. $p_k =$ position; $v_k =$ velocity. (disturbed with process noise)
UAV's objective: UAV wants to land at position [0, 0, 0] with 0 velocity, by controlling its acceleration.

Simulations: robot localization scenario



Available sensors and landmarks for localization:

- 1 GPS (measuring position);
- 1 altimeter;
- 1 stereo camera;
- 10 landmarks on the ground;

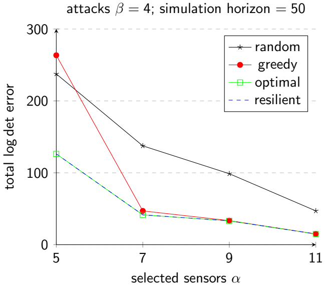
corrupted with measurement noise (GPS, altimeter, stereo camera)

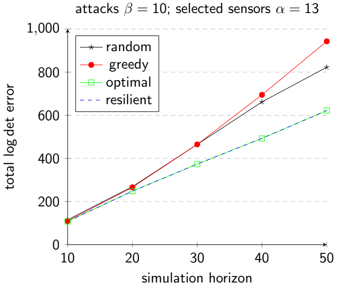
noisy knowledge of their position (landmarks)

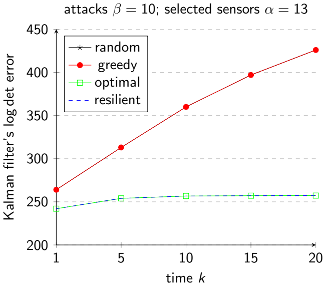
Sensor selection metric: Anticipation error $\log\det[\Sigma(x_k, \ldots, x_{k+20})]$.

minimum mean square error covariance (Σ)

Simulation results







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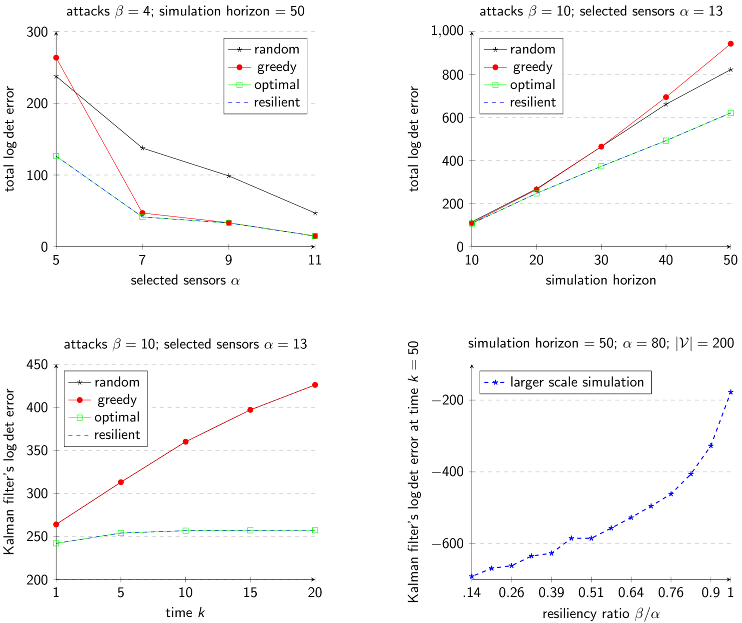

Summary and extensions

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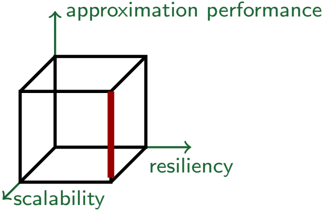

Extensions:

- Matroid constraints;
- Approximatelly submodular functions.